MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

TRIMESTER 3, 2017/2018

FINAL EXAMINATION

EMT2046 – ENGINEERING MATHEMATICS IV

(CE, EE, LE, NE, OPE, TE)

1 JUNE 2018

3:00 PM – 5:00 PM

(2 Hours)

INSTRUCTIONS TO STUDENT

1. This exam paper consists of **NINE (9) pages** (including cover page) with **FOUR (4) questions** and **TWO (2) appendices** only.

2. Attempt **ALL questions**. All questions carry equal marks and the distribution of marks for each question is given.

3. Please print all your answers in the Answer Booklet provided. Show all relevant steps to obtain maximum marks.

4. Only NON-PROGRAMMABLE calculator is allowed.

Question 1

(a) Approximate the positive root of the equation $4800\left(1-e^{-x/10}\right)-320x=0$ by using Newton-Raphson's method with initial value as 8. Use the termination criterion $\left|x_n-x_{n-1}\right|<\varepsilon$, where $\varepsilon=0.0004$.

[10 marks]

(b) Approximate the integral $\int_1^6 2+\sin\left(2\sqrt{x}\right)dx$ by using the composite trapezoidal rule with 8 intervals.

[4.5 marks]

(c) Consider the following system of linear equations:

$$6x_1-2x_2+x_3=11$$
$$x_1+2x_2-5x_3=-1$$
$$-2x_1+7x_2+2x_3=5$$

By using Gauss-Seidel method, estimate $\left[x_1,x_2,x_3\right]$ after two iterations starting from $\left[x_1,x_2,x_3\right]=\left[0,0,0\right]$. Perform necessary row interchange to ensure the solutions converge.

[10.5 marks]

Question 2

(a) Suppose X is a continuous random variable with probability density function:

$$f_X(x)=\frac{3x^2}{16}$$

for $-2<x<2$. What is the probability density function of $y=x^2$?

[9 marks]

(b) A group of statisticians from Town ABC found that weather is a main concern of the population in Town ABC. A study of the town's weather pattern in early spring yields the following observations:

- It is impossible to have two sunny days in a row. But there is a 70% chance that a sunny day is followed by a rainy day.
- If it rains one day, there is a 20% that it is snowing the next day.
- If it snows or rains one day, there is a 50% chance that the weather remains the same the next day.
- It is just as likely for the weather to change from snowy to sunny, as it does from sunny to snowy.

You may use 0, 1 and 2 to represent sunny, rainy and snowy days respectively.

Continued...

(i) Construct the transition probability matrix to model this system.

[5 marks]

(ii) If today is a sunny day, what is the probability that it will snow after three days?

[3 marks]

(iii) Find the chances of the weather being sunny, rainy and snowy after a long run.

[8 marks]

Question 3

(a) Convert the following linear programming (LP) problem to its **dual** problem (but **do not** solve!):

$$\begin{aligned} \text{Maximize } & z = 5x_1 + 6x_2 \\ \text{subject to } & x_1 + 2x_2 = 5 \\ & -x_1 + 5x_2 \geq 3 \\ & x_2 \leq 7 \\ & x_1 \text{ .urs}, x_2 \geq 0 \end{aligned}$$

[7 marks]

(b) HiTec Inc. is a small electronics firm that manufactures low-pass filters and pen drives, the assembly of which require resistors, capacitors, and chips. The following LP model optimizes HiTec's profit based on the availability of resources:

$$\begin{aligned} \text{Maximize } z = \; & 3x_1 + 4x_2 && \text{(profit equation)} \\ \text{subject to: } & 2x_1 + 3x_2 \leq 1500 && \text{(availability of resistors)} \\ & 2x_1 + \; x_2 \leq 1000 && \text{(availability of capacitors)} \\ & x_2 \leq 200 && \text{(availability of chips)} \\ & x_1, x_2 \geq 0 \end{aligned}$$

where x_1 and x_2 are the units of low-pass filters and pen drives to be produced, respectively.

Continued...

(i) Solve the given LP problem using the simplex method to determine the optimum production mix and corresponding maximum profit. Specify the entering and leaving variables, as well as the elementary row operations for each iteration. For the benefit of comparison, a partially-filled simplex tableau at the optimal stage is given as follows:

Basic	z	x_1	x_2	s_1	s_2	s_3	Solution
z	1	0	0	0	?	?	?
s_1	0	0	0	1	–1	–2	?
x_1	0	1	0	0	1/2	–1/2	?
x_2	0	0	1	0	0	1	?

[12 marks]

(ii) For the coming year, HiTec is considering to introduce a new pen drive model which has double the memory capacity of the current model. Each unit of the new product is expected to return a profit of \$5 and its assembly requires 3 resistors, 2 capacitors and 1 chip. Assuming resource availability remains, show how this additional activity modifies the original LP model.

[3 marks]

(iii) In light of the modified LP model in (ii), advise the company on what adjustments (if any) should be made to production levels and how this would impact HiTec's profit. (*Note*: APPENDIX II may be useful for answering this question.)

[3 marks]

Continued...

Question 4

(a) Table Q4a shows the interconnecting distances (in km) between 7 cities in a certain state (the connections are directed from cities of lower to higher number). From the table, draw a graph representing the cities and their interconnections.

[5 marks]

To \ From	City 1	City 2	City 3	City 4	City 5	City 6
City 2	5					
City 3	14					
City 4	6					
City 5		12	3			
City 6		12		9		
City 7					4	4

Table Q4a

Determine the shortest route(s) from City 1 to City 7 using dynamic programming. Summarize your calculations in tableau form.

[5 + 3 marks]

(b) Apply Kruskal's Algorithm to the graph in Figure Q4b to find the minimum spanning tree. Show your solution in table form and draw the resulting spanning tree.

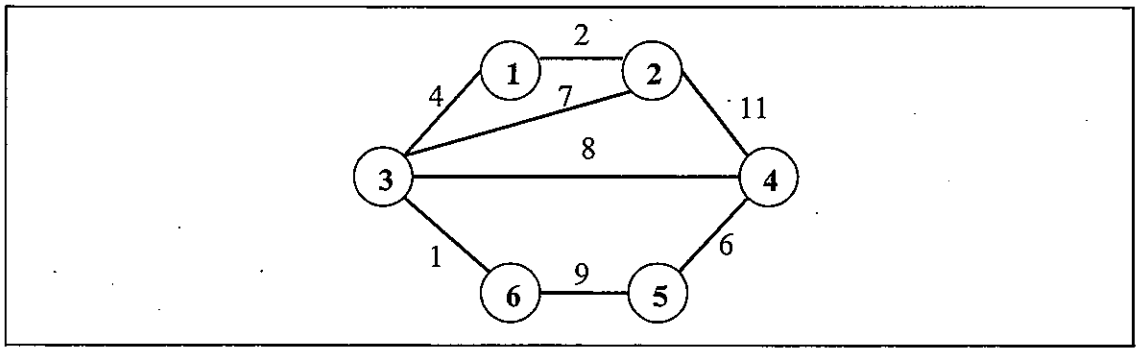

Figure Q4b

[9 + 3 marks]

Continued...

APPENDIX I: List of Formulas

1. The nth Lagrange interpolating polynomial (LIP)

$$f(x) \approx P_n(x) = \sum_{i=0}^{n} L_i(x) f(x_i)$$

with

$$L_k(x) = \prod_{\substack{i=0 \\ i \neq k}}^{n} \frac{(x - x_i)}{(x_k - x_i)}.$$

2. Newton's divided-difference interpolating polynomial (NDDIP)

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} f[x_0, x_1, ..., x_k](x - x_0)\Lambda\ (x - x_{k-1}).$$

3. The error in interpolating polynomial.

$$f(x) - P_n(x) = \frac{(x - x_0)(x - x_1)...(x - x_n)}{(n+1)!} f^{(n+1)}(c_x)$$

for each $x \in [x_0, x_n]$, a number $c_x \in (x_0, x_n)$ exists.

4. Newton's forward-difference formula

$$P_n(x) = f[x_0] + \sum_{k=1}^{n} \binom{s}{k} \Delta^k f(x_0).$$

5. Newton's backward-difference formula

$$P_n(x) = f[x_n] + \sum_{k=1}^{n} (-1)^k \binom{-s}{k} \nabla^k f(x_n).$$

6. Forward difference formula

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}.$$

Backward difference formula

$$f'(x) \approx \frac{f(x) - f(x-h)}{h}.$$

The error term for both forward and backward difference formula is

$$\left| \frac{h}{2} f''(c_x) \right|.$$

Continued...

7. Central difference formula

$$f'(x) \approx \frac{f(x+h)-f(x-h)}{2h}$$

with the error term

$$\left|\frac{h^2}{6} f^{(3)}(c_x)\right|.$$

8. Trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{h}{2}\left(f(a)+f(b)\right).$$

The error term is $\left|\frac{h^3 f''(\xi)}{12}\right|$ for some ξ in (a, b) and $h = b - a$.

9. Composite Trapezoidal rule

$$\int_a^b f(x)dx \approx \frac{h}{2}\left[f(a)+f(b)+2\sum_{j=1}^{n-1} f(x_j)\right].$$

The error term is $\left|\frac{(b-a)h^2 f''(\xi)}{12}\right|$ for some ξ in (a, b) and $h = \frac{b-a}{n}$.

10. Simpson's rule

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a)+4f\left(\frac{a+b}{2}\right)+f(b)\right].$$

The error term is $\left|\frac{h^5}{90} f^{(iv)}(\xi)\right|$ for some ξ in (a, b) and $h = \frac{b-a}{2}$.

11. Composite Simpson's rule

$$\int_a^b f(x)dx \approx \frac{h}{3}\left[f(a)+2\sum_{j=1}^{\left(\frac{n}{2}\right)-1} f(x_{2j})+4\sum_{j=1}^{\frac{n}{2}} f(x_{2j-1})+f(b)\right].$$

The error term is $\left|\frac{(b-a)h^4}{180} f^{(4)}(\xi)\right|$ for some ξ in (a, b), $h = \frac{b-a}{n}$, $x_j = a + jh$, $j = 0,1,\text{K}\ ,n$.

12. Four-term Gaussian Quadrature

$$\int_a^b f(x)dx = \frac{(b-a)}{2}\int_{-1}^{1} f\left(x = \frac{(b+a)+t(b-a)}{2}\right)dt$$

Continued...

$$\int_{-1}^{1} f(x)\,dx \approx w_1 f(x_1) + w_2 f(x_2) + w_3 f(x_3) + w_4 f(x_4)$$

Number of terms n	Nodes x_i	Weights w_i
4	-0.8611363116	0.3478548451
	-0.3399810436	0.6521451549
	0.3399810436	0.6521451549
	0.8611363116	0.3478548451

13. Newton-Raphson's method

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}, \qquad n = 1,2,...$$

14. Euler's method

$$y_{i+1} = y_i + hf(x_i, y_i)$$

with local error $\frac{h^2}{2} Y''(\xi_i)$ for some ξ_i in (x_i, x_{i+1}).

15. Runge Kutta method of order two (Improved Euler method)

$$y_{i+1} = y_i + \tfrac{1}{2}(k_1 + k_2)$$
$$k_1 = hf(x_i, y_i)$$
$$k_2 = hf(x_i + h, y_i + k_1)$$

16. Runge Kutta method of order four

$$k_1 = hf(x_i, y_i),$$
$$k_2 = hf\left(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_1\right),$$
$$k_3 = hf\left(x_i + \tfrac{1}{2}h, y_i + \tfrac{1}{2}k_2\right),$$
$$k_4 = hf(x_{i+1}, y_i + k_3),$$
$$y_{i+1} = y_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4).$$

Continued...

APPENDIX II: Matrix Representation of the Simplex Tableau

Consider a representative LP problem:

$$\text{Maximize} \quad z = \mathbf{c}^T\mathbf{x}$$
$$\text{subject to:} \quad A\mathbf{x} \le \mathbf{b}$$
$$\mathbf{x} \ge \mathbf{0}$$

At any iteration, the matrix representation of the simplex tableau is given by:

Basic	z	$\mathbf{x}$	slack variables	Solution
z	1	$\mathbf{y}^TA - \mathbf{c}^T$	$\mathbf{y}^T = \mathbf{c}_B{}^TB^{-1}$	$\mathbf{y}^T\mathbf{b}$
$\mathbf{x}_B$	**0**	$B^{-1}A$	B^{-1}	$B^{-1}\mathbf{b}$

where $\mathbf{c}^T$ is the row vector of original objective function coefficients,

A is the matrix of original LHS coefficients of the main constraints,

$\mathbf{b}$ is the column vector of the RHS of the main constraints,

$\mathbf{x}_B$ is the column vector of basic variables for the current iteration,

$\mathbf{c}_B{}^T$ is the row vector of original objective coefficients corresponding to $\mathbf{x}_B$,

B is the matrix of original LHS constraint coefficients corresponding to $\mathbf{x}_B$.

End of Paper